

**AFRL-VA-WP-TR-1999-3001**

**AN ALTERNATIVE CLOSED-FORM  
STRESS INTENSITY SOLUTION FOR  
SINGLE PART-THROUGH AND  
THROUGH-THE-THICKNESS CRACKS  
AT OFFSET HOLES**



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**DECEMBER 1998**

**FINAL REPORT FOR 05/15/1998 – 07/13/1998**

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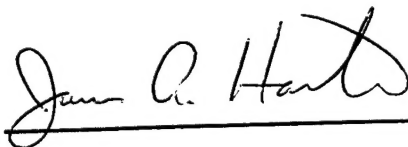
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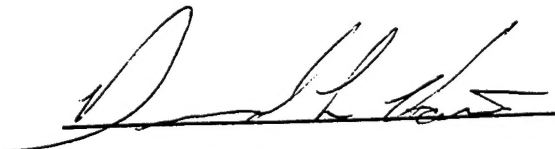
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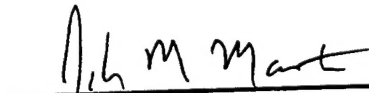
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REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
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1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE DECEMBER 1998		3. REPORT TYPE AND DATES COVERED FINAL REPORT FOR 05/15/1998 - 07/13/1998
4. TITLE AND SUBTITLE AN ALTERNATIVE CLOSED-FORM STRESS INTENSITY SOLUTION FOR SINGLE PART-THROUGH AND THROUGH-THE-THICKNESS CRACKS AT OFFSET HOLES			5. FUNDING NUMBERS PE 62201 PR 2401 TA LE WU 00	
6. AUTHOR(S)  JAMES A. HARTER				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) AFRL/VASE 2790 D STREET, AREA B, BLDG 65 RM 504 WRIGHT-PATTERSON AFB, OH 45433-7542			8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) PROPULSION DIRECTORATE AIR FORCE RESEARCH LABORATORY AIR FORCE MATERIEL COMMAND WRIGHT-PATTERSON AFB, OH 45433-7251 POC: JAMES A. HARTER, AFRL/VASE, 937-255-6104 EXT. 246			10. SPONSORING/MONITORING AGENCY REPORT NUMBER  AFRL-VA-WP-TR-1999-3001	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION AVAILABILITY STATEMENT APPROVED FOR PUBLIC RELEASE, DISTRIBUTION UNLIMITED			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) This report describes work performed to develop a new closed-form stress intensity solution for single cracks at non-centered (offset) holes in plates. The solution is compared to hundreds of finite element and boundary element models and is shown to be in very good agreement.				
14. SUBJECT TERMS Stress Intensity, Structural Integrity, Offset Hole, Fracture Mechanics			15. NUMBER OF PAGES 29	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT  UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE  UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT  UNCLASSIFIED	20. LIMITATION OF ABSTRACT  SAR	

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## FOREWORD

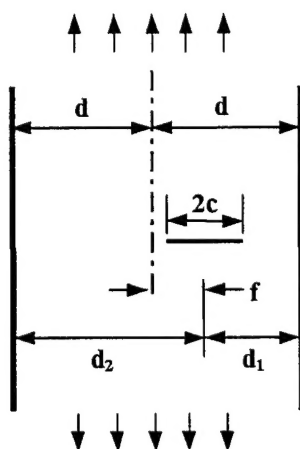
This report summarizes work performed to develop an alternative closed-form stress intensity factor solution for a single crack (*part through or through the thickness*) at an offset hole. The current solution used by many popular crack-growth life prediction programs does not include the effect of crack length. The solution provided in this report includes the effect of crack length and is compared to hundreds of numerical solutions for various offset hole geometries.

The author would like to thank APES, Inc (Dr. Scott Prost-Domasky and Kyle Honeycutt) and NASA/Johnson Space Center (Dr. Royce Forman) for supplying the numerical solutions used to help develop and verify the proposed closed-form solution.

# 1.0 Introduction

## 1.1 Background

Some existing fatigue crack growth life prediction programs use a popular stress intensity solution for a crack growing from a non-centered (offset) hole in a plate that is referenced to a solution for a non-centered through-the-thickness crack given in a publication by Isida [1]. Actually, the solution is really a correction applied to the standard solution for a single crack at a centered hole with a width set equal to twice the distance from the center of the hole to the near edge of the plate. The correction was determined by substituting the hole diameter for the total crack length in the Isida solution. The problem with this solution is the fact that the resulting model makes no provision for the effect of crack length on the stress intensity correction.



**Figure 1: Offset Through-the-Thickness Crack**

Isida [1] states (Appendix III, p. 109) that it is noteworthy that the correction for crack offset is close to that of a centrally cracked strip whose width is twice the distance from the center of the crack to the near edge as long as the eccentricity is not large ( $\delta \leq 0.4$ ). Here, Delta is the distance between the center of the plate to the center of the crack, divided by one half of the plate width.

$$\delta = f / d \quad \text{Equation 1}$$

The reference states that the beta value for a center-cracked plate is:

$$F(\lambda) = \sqrt{\sec\left(\frac{\pi\lambda}{2}\right)} \quad \text{Equation 2}$$

Although the reference does not define lambda directly, it is clear from the well-known center through crack solution (Figure 1.0) that the following is true:



$$\lambda = c/d_1 \quad \text{Equation 3}$$

Where  $c$  is the half crack length and  $d_1$  is the distance from the center of the crack to the near edge of the plate (or one half the plate width ( $W/2$ ) for the center-cracked case)

The solution for the general eccentric (offset) case is given as:

$$F_A(\delta, \lambda) = \sqrt{\sec\left(\frac{\pi\lambda}{2}\right) \frac{\sin(2\lambda\delta)}{2\lambda\delta}} \quad \text{Equation 4}$$

Where the limits of accuracy for the equation are:

$$\lambda \leq 0.7 \sim 0.8 \text{ for all } \delta \text{ values}$$

In short, Isida is saying that the solution to the eccentric internal through-the-thickness crack may be determined by applying a correction factor to the known center crack solution. The plate width is simply set to be twice as wide as the distance from the center of the crack to the nearest plate edge. This correction is expressed as:

$$\sqrt{\frac{\sin(2\lambda\delta)}{(2\lambda\delta)}} \quad \text{Equation 5}$$

## 1.2 Current Offset Hole Solutions

The Isida reference [1] is currently used to determine the stress intensity solution for a single crack at a non-centered (offset) hole using an offset correction factor (in addition to the finite width effect,  $F_w$ ) as follows:

$$K_{(\text{offset})} = K_{(\text{infinite plate})} * F_w * F_{\text{offset}} \quad \text{Equation 6}$$

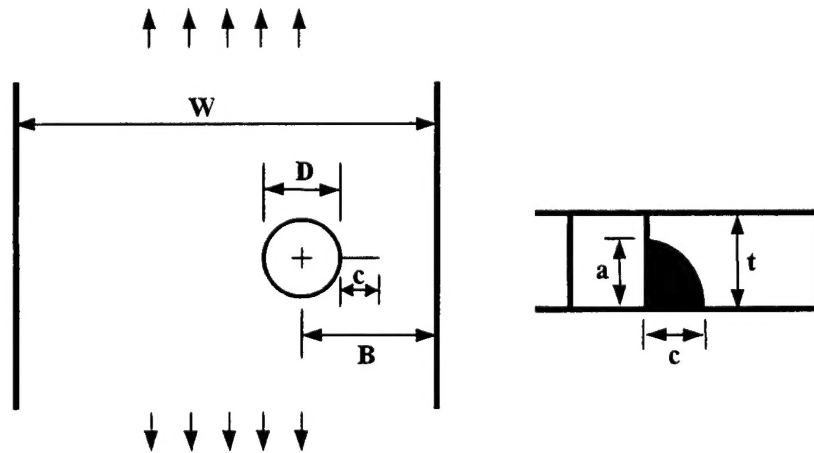


Figure 2: Single Crack at an Offset Hole

For a single cracked hole in a plate (part-through or through):

$$F_w = \sqrt{\sec\left(\frac{\pi}{2} \sqrt{\frac{a}{t} \left(\frac{D+c}{2B-c}\right)}\right) \sec\left(\frac{\pi D}{4B}\right)} \quad \text{Equation 7}$$

$$F_{\text{offset}} = \sqrt{\frac{\sin(\gamma)}{\gamma}} \quad \text{Equation 8}$$

Note: For a through crack,  $\left(\frac{a}{t}\right)$  is assumed to be equal to 1.0.

Equation 7 ( $F_w$ ) is the actual width correction for a single crack at a hole located in the center of a plate (width = 2B). The square root of the first secant term is widely used as the finite width correction for a single through-cracked, centered hole (again, with  $a/t = 1.0$ ). The second secant term in the radical is referenced by Dr. James Newman [2] for a single part through cracked, centered hole. Discussions with Dr. Newman and comparisons with finite element results strongly suggest that this term should be included for BOTH part through and through-the-thickness cracks. This will be shown later in this report.

In these solutions, gamma ( $\gamma$ ) is currently defined as:

$$\gamma = \frac{D}{B} - \frac{2D}{W} \quad \text{Equation 9}$$

### 1.3 Problem with the Current Solution

It seems clear that gamma is equivalent to  $[2\lambda\delta]$  in the Isida reference [1]. This may be shown by assuming that the developers of these solutions substituted the offset hole in place of the internal crack (without a hole) in Isida's equation as follows:

$$\lambda = \frac{D}{2B} \quad \text{Equation 10}$$

$$\delta = \frac{(W - 2B)}{W} \quad \text{Equation 11}$$

Examine the following:

**Table 1: Assumptions in Current Offset Hole Solution**

Offset Hole Parameter	Equivalent to Isida Internal Crack Parameter
D	2c
B	d1
$\gamma$	$2\lambda\delta$

By substitution:

$$\gamma = 2\lambda\delta = 2 \frac{D}{2B} \frac{(W-2B)}{W} = \frac{D}{B} \frac{(W-2B)}{W} = \frac{D}{B} - \frac{2D}{W} \quad \text{Equation 12}$$

Therefore, gamma is equivalent to Isida's value ( $2\lambda\delta$ ) IF you replace the internal through crack in Isida's equation with the hole (neglecting the crack)!

Here is where the problem develops. First, a hole is not a crack. Isida's correction is a function of crack length and the use of the hole dimensions alone will insure a constant correction value for all crack lengths

**Table 2: Offset Correction Values**

$\gamma$	$\sqrt{\frac{\sin(\gamma)}{\gamma}}$
0.001	1.000000
0.010	0.999992
0.050	0.999792
0.100	0.999167
0.200	0.996668
0.300	0.992506
0.400	0.986684
0.500	0.979209
0.750	0.953337
1.000	0.917317
1.500	0.815473
2.000	0.674276
2.500	0.489274
3.000	0.216887
$\pi$	0.000000

It seems logical that Isida's correction would account for a reduction in stress intensity from what would have been predicted assuming the plate width was simply twice the distance from the crack center to the near edge. As the crack grows, the load will tend to shift to the non-cracked (far) side. It is also clear that gamma can never exceed pi.

## 2.0 Proposed Solution

The effect of crack length on the Isida correction may be determined by analogy to the Isida reference using the value of lambda that is appropriate for the crack geometry. The hole offset effect is included in the finite width correction for a given crack model. The complete stress intensity solution is the product of the infinite plate solution and the finite width effect.

It is proposed that the current gamma value be amended as follows:

$$\gamma = 2\lambda\delta \quad \text{Equation 13}$$

The value of lambda ( $\lambda$ ) is based on the finite width correction (Equation 7) and by analogy (see Equation 4) as follows:

Single Part-Through-the-Thickness Crack at a Hole:

$$\lambda = \sqrt{\frac{a}{t} \left( \frac{D+c}{2B-c} \right)} \quad \text{Equation 14}$$

Single Through-the-Thickness Crack at a Hole ( $a/t = 1$ ):

$$\lambda = \left( \frac{D+c}{2B-c} \right) \quad \text{Equation 15}$$

Again, the plate width has been adjusted to be twice the near edge distance ( $2B$ ) in the above equations and the finite width effect,  $F_w$  (Equation 7). The second term,  $\sqrt{\sec\left(\frac{\pi D}{4B}\right)}$ , in the finite width effect (Equation 7) accounts for the effect of the hole itself (without a crack) in the stress intensity solution. This term has been ignored for the purposes of the analogy with the Isida solution.

The delta parameter remains unchanged (see Equation 11).

The proposed offset hole correction given below is based on the above analogy as well as a large number of finite and boundary element models [3, 4].

$$F_{\text{offset}} = \frac{\sin\left(\sqrt{\frac{a}{t} \left( \frac{D+c}{B-c/2} \right) \left( \frac{W-2B}{W} \right)}\right)}{\sqrt{\frac{a}{t} \left( \frac{D+c}{B-c/2} \right) \left( \frac{W-2B}{W} \right)}} \left( 1 - \left( \frac{c}{(B-D/2)} \right)^{12} \right)^{\frac{D}{2B}} \quad \text{Equation 16}$$

It should be noted that the proposed solution differs from what would be expected from Isida's offset correction (Equation 8) since the entire correction is NOT inside a radical. It was discovered that the correction matched the finite and boundary element cracked hole solutions more closely without the radical. An additional term was also required to provide a better fit to the computational models at longer crack lengths. This additional term is given in terms of the fraction of the distance from the hole to the near edge ( $c/(B-D/2)$ ).

### 3.0 Example Case

The proposed offset correction is shown below for the following geometry:

Single Through-the-Thickness Crack at an Offset Hole in a Plate

Width = 4.0 in.

Thickness = 0.25 in.

Hole Diameter = 0.25 in.

B (Offset Distance) = 1.0 in.

**Table 3: Comparison of the Current and Proposed Corrections ( $F_{offset}$ )**

Crack Length (c)	$c/(B-D/2)$	Current Correction	Proposed Correction	Percent Change
0.001	0.0011	0.9987	0.9974	0.133
0.010	0.0114	0.9987	0.9972	0.154
0.030	0.0343	0.9987	0.9966	0.206
0.050	0.0571	0.9987	0.9961	0.264
0.100	0.1143	0.9987	0.9944	0.435
0.200	0.2286	0.9987	0.9896	0.909
0.250	0.2857	0.9987	0.9864	1.226
0.375	0.4286	0.9987	0.9755	2.321
0.500	0.5714	0.9987	0.9587	4.004
0.750	0.8571	0.9987	0.8777	12.114
0.850	0.9714	0.9987	0.7331	26.599

Although the change in the correction ( $F_{offset}$ ) is not large until the crack length is relatively long, the proposed solution appears to be consistent with the Isida reference (without the radical) and is a function of crack length. In addition, the proposed stress intensity solution includes a change to the finite width effect for the through-the-thickness crack to include the secant term for the hole without a crack and a plate width equal to 2B (see Equation 7). This results in a higher value for the stress intensity and is also consistent with the other assumptions that are being made.

## 4.0 Comparison with Finite and Boundary Element Results

A large number of Boundary Element Method (**BEM**) and Finite Element Method (**FEM**) analyses were performed [3, 4] and the results were provided for comparison with the proposed closed form solution. Over 200 BEM and nearly 100 FEM analyses were compared to the proposed solution. All of these comparisons were performed for a single through-the-thickness crack at an offset hole. The results are given in the following sections.

### 4.1 Centered Hole Results

The single crack, centered hole solution was compared to the BEM (FADD computer code [3]) analytical results to act as a control sample to show how this standard, widely accepted closed-form solution performs. The closed-form solution in AFGROW uses the single, through cracked hole in an infinite plate from Tada, Paris, and Irwin [5] and the finite width correction from Newman [2]. The Newman single, through cracked hole solution [6] differs in the equation fit used for the infinite plate portion of the solution. It should be noted that the closed form solution diverges from the BEM solution at high hole diameter to plate width ratios.

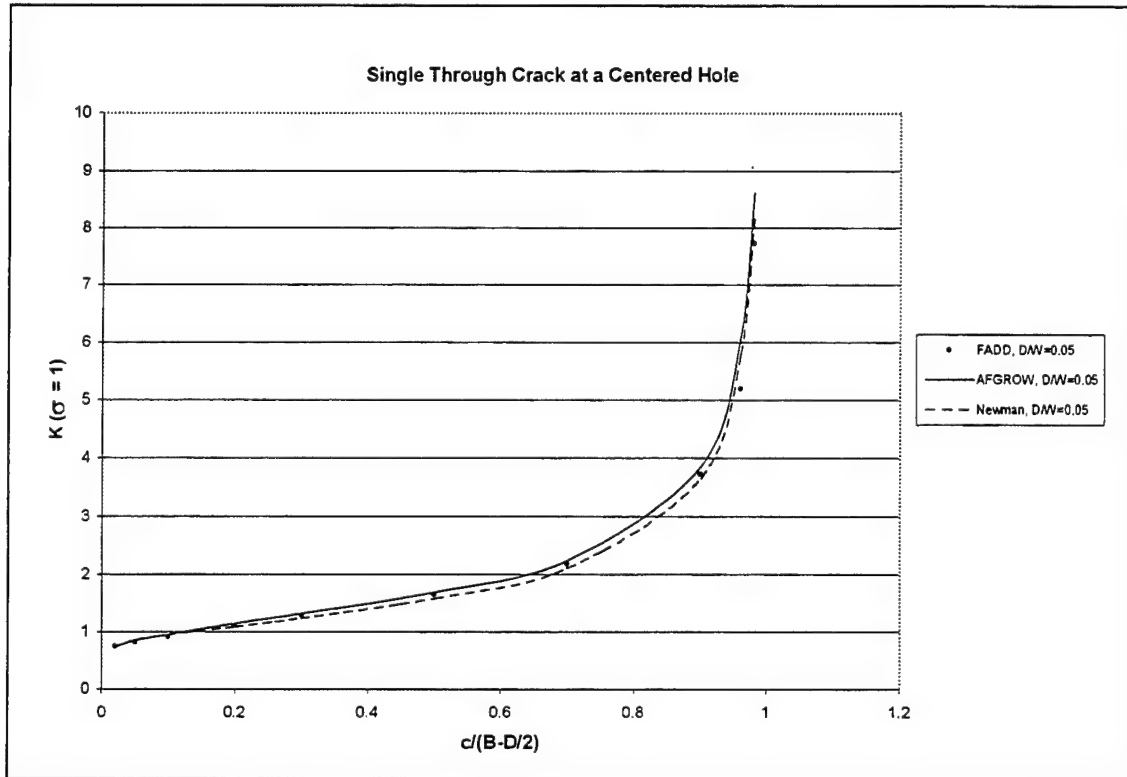


Figure 3: Single Through Crack at a Centered Hole ( $D/W = 0.05$ )

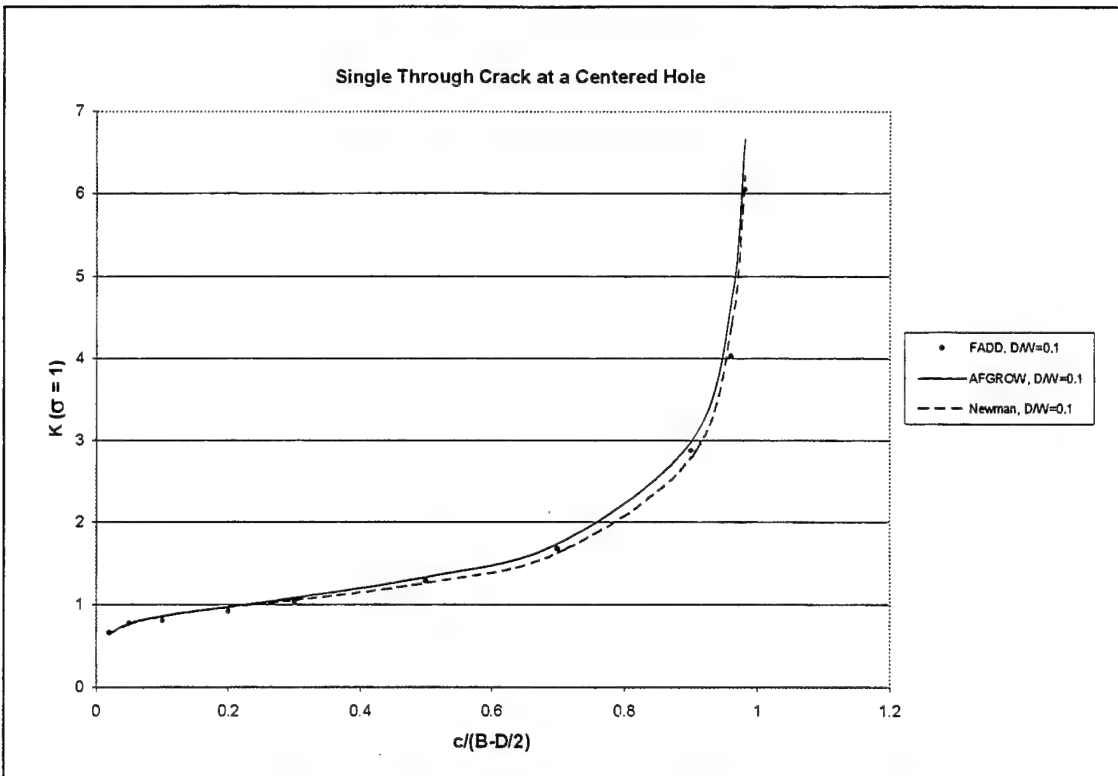


Figure 4: Single Through Crack at a Centered Hole ( $D/W = 0.1$ )

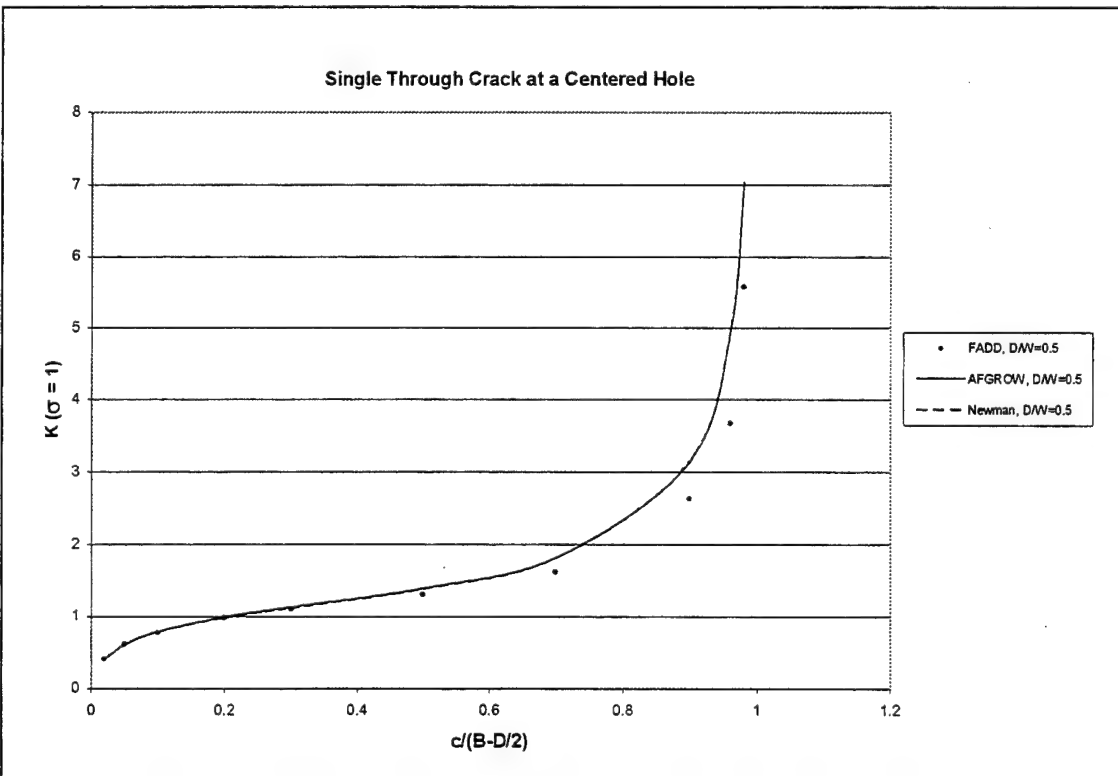


Figure 5: Single Through Crack at a Centered Hole ( $D/W = 0.5$ )



## 4.2 Offset Hole Results

The offset hole results for the proposed solution were compared to the BEM (FADD computer code) and the FEM (STRESSCHECK computer code [4]) for several through crack geometries. The comparisons for the centered hole cases shown in paragraph 4.1 are important since the offset hole solution uses the centered hole solution for  $W=2B$  and applies the appropriate offset correction (Equation 16). The closed form offset hole solution proposed in Equation 16 is used in AFGROW and is compared to the BEM and FEM results. The offset hole correction is generally somewhat conservative for longer crack lengths ( $c/(B-D/2) > 0.8$ ). Since the centered hole solution is noticeably conservative for  $D/W = 0.5$  and  $c/(B-D/2) > 0.5$ , this will be reflected in the comparison for  $D/2B = 0.5$ .

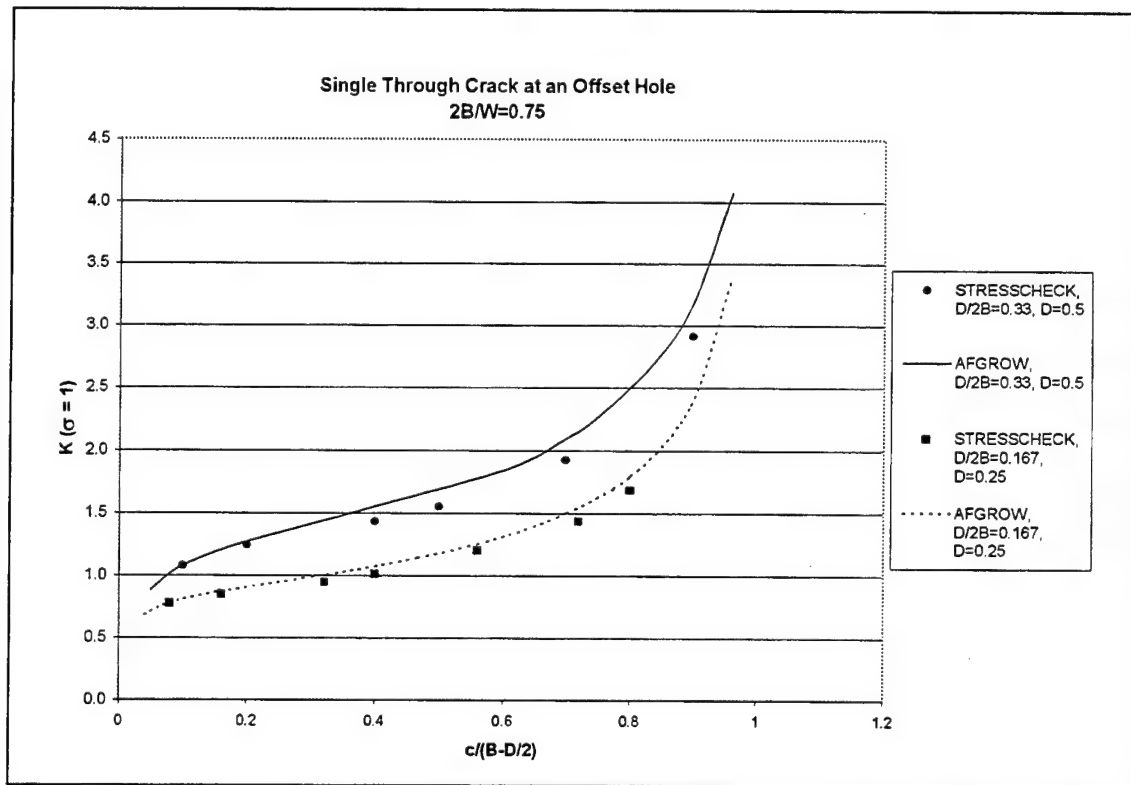
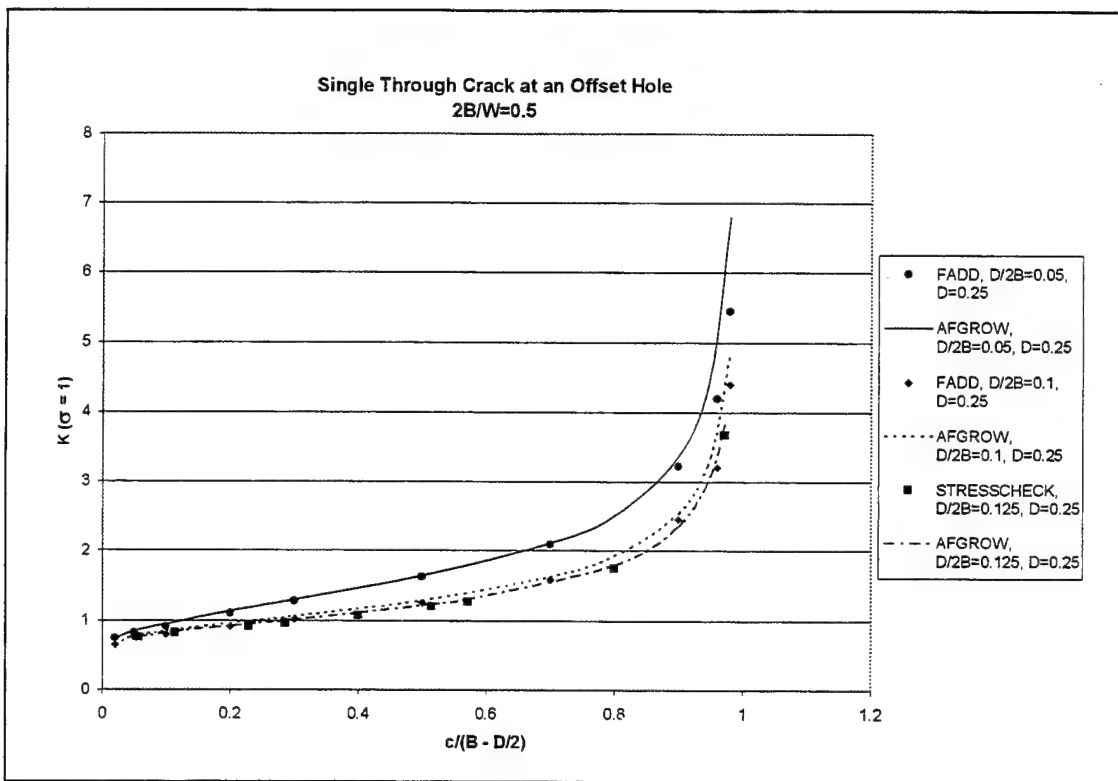
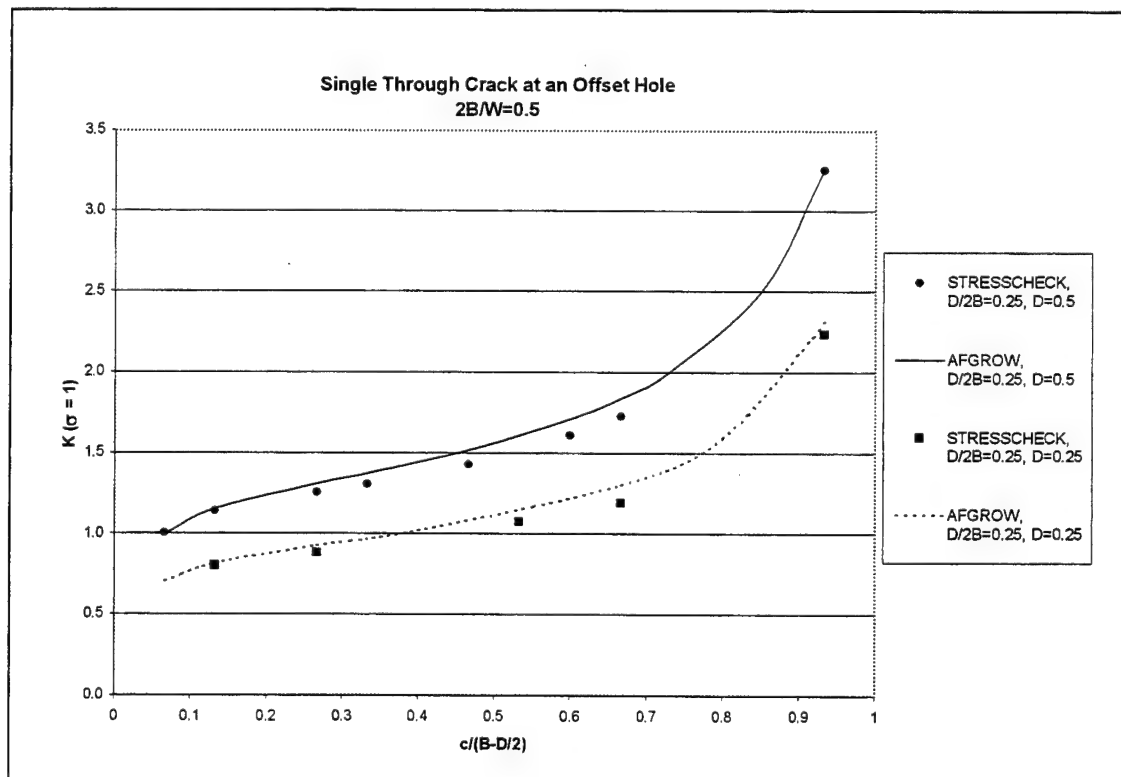


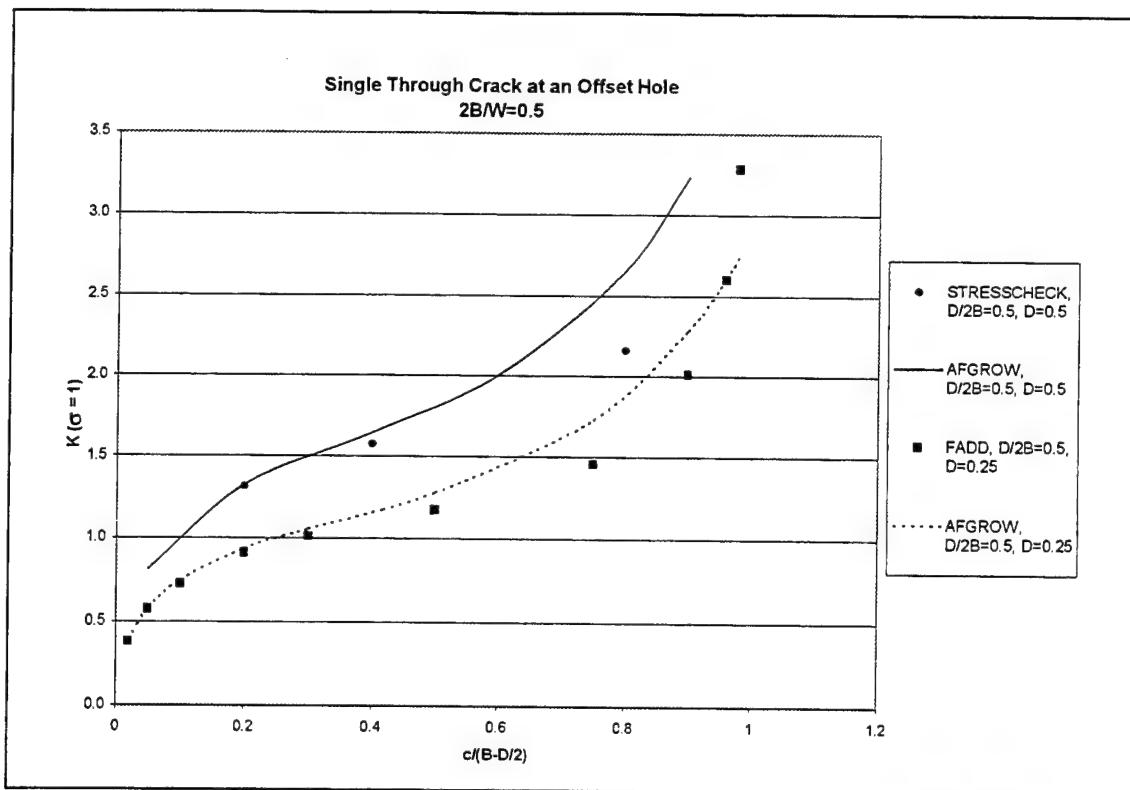
Figure 6: Through Crack at an Offset Hole ( $2B/W = 0.75$ ,  $D/2B = 0.167 - 0.33$ )



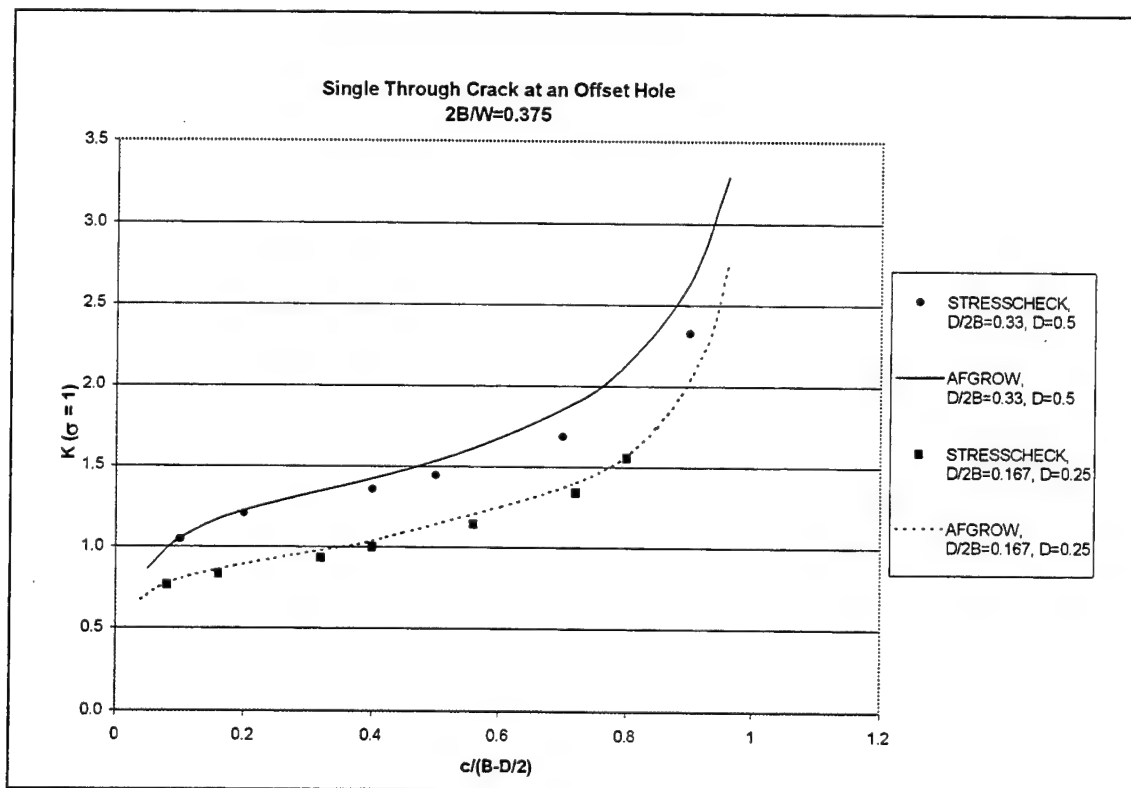
**Figure 7: Through Crack at an Offset Hole ( $2B/W = 0.5$ ,  $D/2B = 0.125$ )**



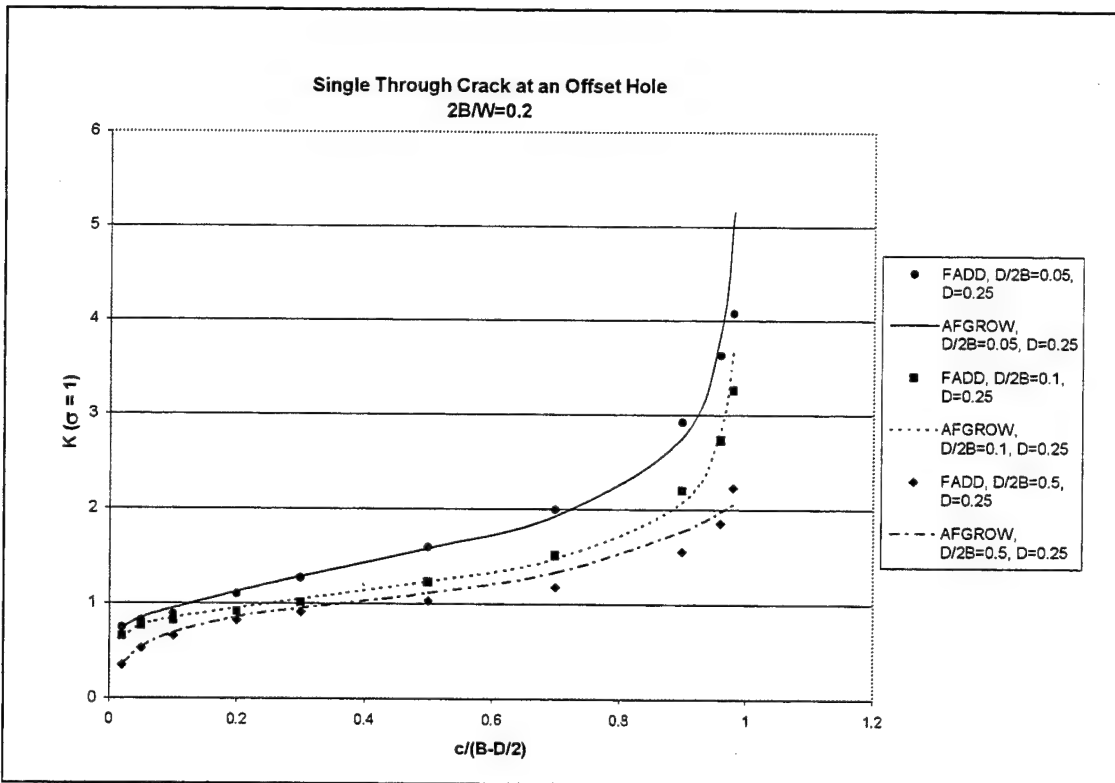
**Figure 8: Through Crack at an Offset Hole ( $2B/W = 0.5$ ,  $D/2B = 0.25$ )**



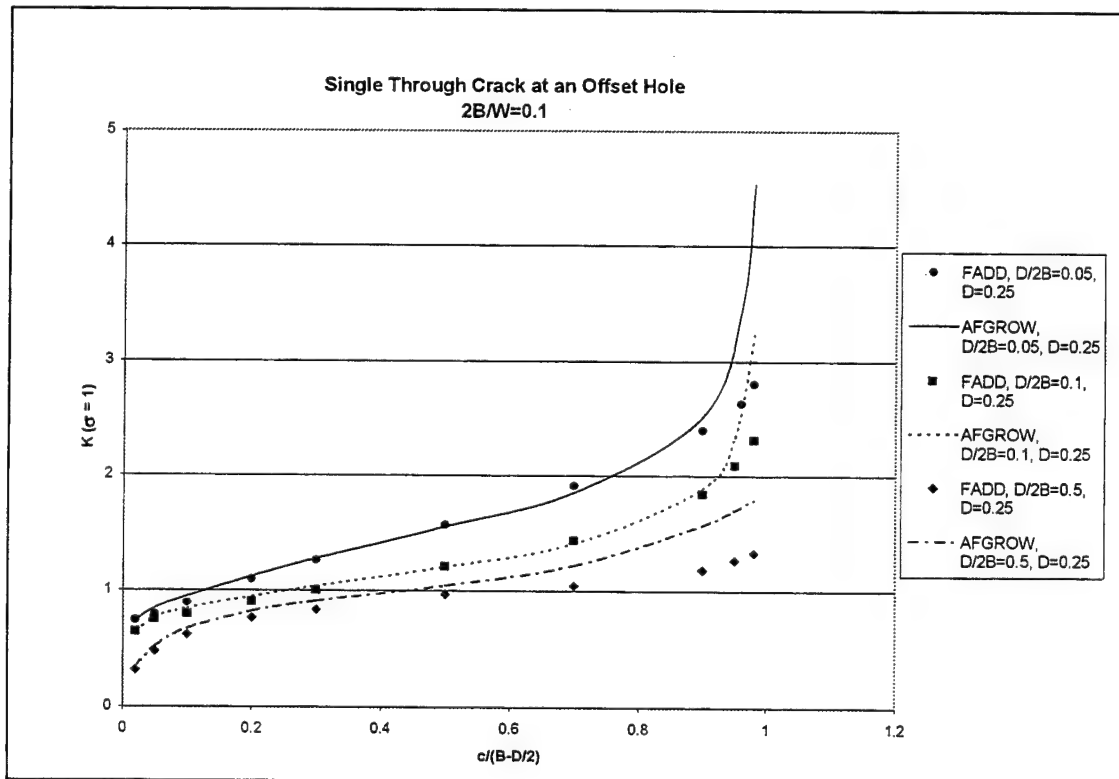
**Figure 9: Through Crack at an Offset Hole ( $2B/W = 0.5$ ,  $D/2B = 0.5$ )**



**Figure 10: Through Crack at an Offset Hole ( $2B/W = 0.375$ ,  $D/2B = 0.167 - 0.33$ )**



**Figure 11: Through Crack at an Offset Hole ( $2B/W = 0.2$ ,  $D/2B = 0.05 - 0.5$ )**



**Figure 12: Through Crack at an Offset Hole ( $2B/W = 0.1$ ,  $D/2B = 0.05 - 0.5$ )**

## 5.0 Conclusions

The proposed closed-form stress intensity solution for single cracks at offset holes shows very good correlation to detailed BEM and FEM computer models. This solution also maintains a relationship between crack length and offset correction that makes physical sense. It is noted that this solution tends to be somewhat conservative for cases of relatively high  $D/2B$  ( $> 0.25$ ) as the crack extends past one half of the distance from the edge of the hole to the near edge of the plate ( $c/(B-D/2)$ ). As the value of  $D/2B$  increases, the conservatism also increases. This is due, in part, to the problem observed in the center hole solution at  $D/2B = 0.5$  and the fact that the offset correction is probably only good for:

$$\lambda = \left( \frac{D+c}{2B-c} \right) \leq 0.7 \quad (\text{Equation 15 and the limits for Equation 4})$$

The problem with the center hole cases for  $D/2B \geq 0.5$  is seen in both the Newman and Tada references [2,5]. There may be a problem in these solutions or perhaps a problem with the computational models [3,4]. In either case, any errors in cases of high  $D/2B$  should not be of tremendous concern since most practical problems will generally have lower  $D/2B$  values. For cases with high values of  $D/2B$ , the crack growth life for a crack approximately  $\frac{1}{2}$  through the near edge will be short indeed.

The addition of the second secant term in the finite width correction ( $F_w$ ) is also a significant part of the proposed solution (Equation 7). This term was proposed by Newman [2] and was included in all of the comparisons shown in this report. This term is not a function of crack length and would not explain the difference in the solutions at high values of  $D/2B$  since the solution shows very close agreement with the computational models at the shorter crack lengths.

## 6.0 REFERENCES

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## Appendix A: Tabular Data Used for the Centered Hole Cases

### Case 1:

**Center Hole Data: Width: 5 in. Hole Diameter: 0.25 in. (W/D=0.05)**

$C$	$C/(B-D/2)$	$F_{\text{c}}(\text{Newman})$	$K(\text{FADD})$	$K(\text{AFGROW})$	$K(\text{Newman})$
0.0475	0.02	1.0038	0.7421	0.7434	0.7345
0.1188	0.05	1.0051	0.8171	0.8496	0.8530
0.2375	0.10	1.0081	0.9097	0.9535	0.9478
0.4750	0.20	1.0177	1.0984	1.1361	1.0833
0.7125	0.30	1.0339	1.2766	1.3108	1.2288
1.1875	0.50	1.0996	1.6437	1.6827	1.5731
1.6625	0.70	1.2704	2.1727	2.2304	2.0991
2.1375	0.90	1.9729	3.7230	3.8584	3.6571
2.2800	0.96	3.0266	5.1889	6.0894	5.7828
2.3275	0.98	4.2396	7.7215	8.6078	8.1795

### Case 2:

**Center Hole Data: Width: 2.5 in. Hole Diameter: 0.25 in. (W/D=0.1)**

$C$	$C/(B-D/2)$	$F_{\text{c}}(\text{Newman})$	$K(\text{FADD})$	$K(\text{AFGROW})$	$K(\text{Newman})$
0.0225	0.02	1.0138	0.6544	0.6458	0.6406
0.0563	0.05	1.0161	0.7676	0.7730	0.7648
0.1125	0.10	1.0208	0.8001	0.8564	0.8591
0.2250	0.20	1.0342	0.9147	0.9682	0.9646
0.3375	0.30	1.0546	1.0253	1.0762	1.0482
0.5625	0.50	1.1315	1.2771	1.3285	1.2559
0.7875	0.70	1.3208	1.6726	1.7336	1.6217
1.0125	0.90	2.0740	2.8695	2.9842	2.7866
1.0800	0.96	3.1916	4.0268	4.7069	4.3965
1.1025	0.98	4.4750	6.0386	6.6523	6.2146

### Case 3:

**Center Hole Data: Width: 0.5 in. Hole Diameter: 0.25 in. (W/D=0.5)**

$C$	$C/(B-D/2)$	$F_{\text{c}}(\text{Newman})$	$K(\text{FADD})$	$K(\text{AFGROW})$	$K(\text{Newman})$
0.0025	0.02	1.4227	0.4073	0.4041	0.4063
0.0063	0.05	1.4361	0.6113	0.6035	0.6052
0.0125	0.10	1.4604	0.7736	0.7875	0.7861
0.0250	0.20	1.5174	0.9779	0.9912	0.9821
0.0375	0.30	1.5892	1.0974	1.1294	1.1156
0.0625	0.50	1.8054	1.3014	1.3953	1.3826
0.0875	0.70	2.2405	1.6115	1.8158	1.8118
0.1125	0.90	3.7388	2.6333	3.1367	3.1466
0.1200	0.96	5.8499	3.6678	4.9524	4.9730
0.1225	0.98	8.2449	5.5755	7.0003	7.0313

## Appendix B: Tabular Data Used for the Offset Hole Cases

### Case 1:

#### STRESSCHECK Comparison: 2B/W: 0.75, D/2B: 0.33

Dia	W	B	2B/W	D/2B	
0.5	2	0.75	0.75	0.33	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (STRESSCHECK)	K (AFGROW)
0.0250	0.05	1.1671	0.9947		0.8854
0.0500	0.10	1.1811	0.9940	1.0783	1.0846
0.1000	0.20	1.2153	0.9924	1.2419	1.2738
0.2000	0.40	1.3196	0.9880	1.4344	1.5218
0.2500	0.50	1.4016	0.9850	1.5519	1.6624
0.3500	0.70	1.7024	0.9729	1.9267	2.0987
0.4500	0.90	2.7834	0.8650	2.9143	3.1879
0.4800	0.96	4.3309	0.7012		4.0707

### Case 2:

#### STRESSCHECK Comparison: 2B/W: 0.75, D/2B: 0.167

Dia	W	B	2B/W	D/2B	
0.25	2	0.75	0.75	0.167	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (STRESSCHECK)	K (AFGROW)
0.0250	0.04	1.0399	0.9986		0.6783
0.0500	0.08	1.0452	0.9982	0.7716	0.7792
0.1000	0.16	1.0586	0.9974	0.8443	0.8715
0.2000	0.32	1.0999	0.9950	0.9472	1.0029
0.2500	0.40	1.1312	0.9933	1.0113	1.0737
0.3500	0.56	1.2316	0.9885	1.2042	1.2533
0.4500	0.72	1.4389	0.9784	1.4375	1.5478
0.5000	0.80	1.6448	0.9652	1.6866	1.7989
0.6000	0.96	3.4465	0.8224		3.3959



**Case 3:**

**FADD Comparison: 2B/W: 0.5, D/2B: 0.05**

Dia	W	B	2B/W	D/2B	
0.25	10	2.5	0.5	0.05	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (FADD)	K (AFGROW)
0.0475	0.02	1.0038	0.9994	0.7414	0.7430
0.1188	0.05	1.0051	0.9990	0.8160	0.8488
0.2375	0.10	1.0081	0.9983	0.9083	0.9519
0.4750	0.20	1.0177	0.9957	1.0953	1.1313
0.7125	0.30	1.0339	0.9916	1.2710	1.2998
1.1875	0.50	1.0996	0.9765	1.6215	1.6431
1.6625	0.70	1.2704	0.9455	2.0836	2.1088
2.1375	0.90	1.9729	0.8734	3.2208	3.3700
2.2800	0.96	3.0266	0.8220	4.1938	5.0054
2.3275	0.98	4.2396	0.7890	5.4447	6.7919

**Case 4:**

**FADD Comparison: 2B/W: 0.5, D/2B: 0.1**

Dia	W	B	2B/W	D/2B	
0.25	5	1.25	0.5	0.1	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (FADD)	K (AFGROW)
0.0225	0.02	1.0138	0.9980	0.6523	0.6445
0.0563	0.05	1.0161	0.9974	0.7647	0.7710
0.1125	0.10	1.0208	0.9962	0.7969	0.8531
0.2250	0.20	1.0342	0.9928	0.9097	0.9612
0.3375	0.30	1.0546	0.9877	1.0176	1.0630
0.5625	0.50	1.1315	0.9709	1.2530	1.2899
0.7875	0.70	1.3208	0.9386	1.5878	1.6272
1.0125	0.90	2.0740	0.8553	2.4596	2.5524
1.0800	0.96	3.1916	0.7822	3.2080	3.6819
1.1025	0.98	4.4750	0.7299	4.4018	4.8554

**Case 5:**

**STRESSCHECK Comparison: 2B/W: 0.5, D/2B: 0.125**

Dia	W	B	2B/W	D/2B	
0.25	4	1	0.5	0.125	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (STRESSCHECK)	K (AFGROW)
0.0500	0.06	1.0247	0.9961	0.7563	0.7623
0.1000	0.11	1.0314	0.9944	0.8165	0.8465
0.2000	0.23	1.0505	0.9896	0.9103	0.9527
0.2500	0.29	1.0638	0.9864	0.9566	1.0027
0.3500	0.40	1.1009	0.9781	1.0666	1.1085
0.4500	0.51	1.1592	0.9663	1.1908	1.2315
0.5000	0.57	1.2008	0.9587	1.2620	1.3045
0.7000	0.80	1.5762	0.9053	1.7410	1.7982
0.8500	0.97	3.8653	0.7331	3.6731	3.8215

**Case 6:**

**STRESSCHECK Comparison: 2B/W: 0.5, D/2B: 0.25**

Dia	W	B	2B/W	D/2B	
0.5	4	1	0.5	0.25	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (STRESSCHECK)	K (AFGROW)
0.0500	0.07	1.0946	0.9868	1.0049	0.9978
0.1000	0.13	1.1094	0.9835	1.1378	1.1524
0.2000	0.27	1.1495	0.9750	1.2544	1.3083
0.2500	0.33	1.1766	0.9697	1.3041	1.3739
0.3500	0.47	1.2524	0.9563	1.4288	1.5176
0.4500	0.60	1.3765	0.9380	1.6075	1.7096
0.5000	0.67	1.4713	0.9258	1.7273	1.8406
0.5500	0.73	1.6056	0.9093		2.0118
0.6400	0.85	2.0751	0.8519		2.5192
0.7000	0.93	2.9966	0.7484	3.2527	3.2648

**Case 7:**

**STRESSCHECK Comparison: 2B/W: 0.5, D/2B: 0.25**

Dia	W	B	2B/W	D/2B	
0.25	2	0.5	0.5	0.25	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (STRESSCHECK)	K (AFGROW)
0.0250	0.07	1.0946	0.9868		0.7055
0.0500	0.13	1.1094	0.9835	0.8024	0.8148
0.1000	0.27	1.1495	0.9750	0.8841	0.9251
0.2000	0.53	1.3062	0.9480	1.0747	1.1348
0.2500	0.67	1.4713	0.9258	1.1888	1.3015
0.3500	0.93	2.9966	0.7484	2.2352	2.3086

**Case 8:**

**STRESSCHECK Comparison: 2B/W: 0.5, D/2B: 0.5**

Dia	W	B	2B/W	D/2B	
0.5	2	0.5	0.5	0.5	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (STRESSCHECK)	K (AFGROW)
0.0125	0.05	1.4361	0.9557		0.8156
0.0500	0.20	1.5174	0.9451	1.3135	1.3247
0.1000	0.40	1.6818	0.9275	1.5767	1.6476
0.2000	0.80	2.6924	0.8465	2.1607	2.6605
0.2250	0.90	3.7388	0.7288		3.2331

**Case 9:**

**FADD Comparison: 2B/W: 0.5, D/2B: 0.5**

Dia	W	B	2B/W	D/2B	
0.25	1	0.25	0.5	0.5	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (FADD)	K (AFGROW)
0.0025	0.02	1.4227	0.9576	0.3825	0.3870
0.0063	0.05	1.4361	0.9557	0.5738	0.5767
0.0125	0.10	1.4604	0.9524	0.7248	0.7500
0.0250	0.20	1.5174	0.9451	0.9108	0.9367
0.0375	0.30	1.5892	0.9368	1.0133	1.0580
0.0625	0.50	1.8054	0.9170	1.1752	1.2795
0.0938	0.75	2.4309	0.8707	1.4595	1.7318
0.1125	0.90	3.7388	0.7288	2.0142	2.2861
0.1200	0.96	5.8499	0.5286	2.6019	2.6176
0.1225	0.98	8.2449	0.3923	3.2855	2.7460

**Case 10:**

**STRESSCHECK Comparison: 2B/W: 0.375, D/2B: 0.33**

Dia	W	B	2B/W	D/2B	
0.5	4	0.75	0.375	0.33	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (STRESSCHECK)	K (AFGROW)
0.0250	0.05	1.1671	0.9673		0.8610
0.0500	0.10	1.1811	0.9630	1.0433	1.0507
0.1000	0.20	1.2153	0.9529	1.2049	1.2231
0.2000	0.40	1.3196	0.9262	1.3579	1.4267
0.2500	0.50	1.4016	0.9088	1.4448	1.5338
0.3500	0.70	1.7024	0.8597	1.6864	1.8545
0.4500	0.90	2.7834	0.7163	2.3261	2.6396
0.4800	0.96	4.3309	0.5659		3.2853

**Case 11:**

**STRESSCHECK Comparison: 2B/W: 0.375, D/2B: 0.167**

Dia	W	B	2B/W	D/2B	
0.25	4	0.75	0.375	0.167	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (STRESSCHECK)	K (AFGROW)
0.0250	0.04	1.0399	0.9910		0.6731
0.0500	0.08	1.0452	0.9889	0.7635	0.7719
0.1000	0.16	1.0586	0.9838	0.8293	0.8596
0.2000	0.32	1.0999	0.9691	0.9301	0.9768
0.2500	0.40	1.1312	0.9588	0.9977	1.0364
0.3500	0.56	1.2316	0.9305	1.1414	1.1797
0.4500	0.72	1.4389	0.8853	1.3404	1.4006
0.5000	0.80	1.6448	0.8497	1.5536	1.5836
0.5300	0.85	1.8489	0.8193		1.7466
0.6000	0.96	3.4465	0.6688		2.7618

**Case 12:**

**FADD Comparison: 2B/W: 0.2, D/2B: 0.05**

Dia	W	B	2B/W	D/2B	
0.25	25	2.5	0.2	0.05	
c	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (FADD)	K (AFGROW)
0.0475	0.02	1.0038	0.9985	0.7396	0.7423
0.1188	0.05	1.0051	0.9976	0.8149	0.8476
0.2375	0.10	1.0081	0.9955	0.8853	0.9493
0.4750	0.20	1.0177	0.9891	1.0928	1.1237
0.7125	0.30	1.0339	0.9786	1.2629	1.2828
1.1875	0.50	1.0996	0.9404	1.5910	1.5824
1.6625	0.70	1.2704	0.8651	1.9919	1.9294
2.1375	0.90	1.9729	0.7165	2.9199	2.7647
2.2800	0.96	3.0266	0.6386	3.6289	3.8888
2.3275	0.98	4.2396	0.5999	4.0764	5.1639

**Case 13:**

**FADD Comparison: 2B/W: 0.2, D/2B: 0.1**

Dia	W	B	2B/W	D/2B	
0.25	12.5	1.25	0.2	0.1	
c	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (FADD)	K (AFGROW)
0.0225	0.02	1.0138	0.9948	0.6502	0.6424
0.0563	0.05	1.0161	0.9933	0.7619	0.7678
0.1125	0.10	1.0208	0.9902	0.8164	0.8480
0.2250	0.20	1.0342	0.9815	0.9079	0.9503
0.3375	0.30	1.0546	0.9688	1.0080	1.0426
0.5625	0.50	1.1315	0.9266	1.2211	1.2310
0.7875	0.70	1.3208	0.8494	1.5068	1.4725
1.0125	0.90	2.0740	0.6963	2.1980	2.0778
1.0800	0.96	3.1916	0.6053	2.7331	2.8492
1.1025	0.98	4.4750	0.5537	3.2556	3.6835

**Case 14:**

**FADD Comparison: 2B/W: 0.2, D/2B: 0.5**

Dia	W	B	2B/W	D/2B	
0.25	2.5	0.25	0.2	0.5	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (FADD)	K (AFGROW)
0.0025	0.02	1.4227	0.8937	0.3478	0.3612
0.0063	0.05	1.4361	0.8890	0.5226	0.5365
0.0125	0.10	1.4604	0.8808	0.6587	0.6936
0.0250	0.20	1.5174	0.8630	0.8241	0.8554
0.0375	0.30	1.5892	0.8431	0.9101	0.9522
0.0625	0.50	1.8054	0.7960	1.0299	1.1107
0.0875	0.70	2.2405	0.7328	1.1756	1.3305
0.1125	0.90	3.7388	0.5644	1.5503	1.7703
0.1200	0.96	5.8499	0.3994	1.8559	1.9781
0.1225	0.98	8.2449	0.2939	2.2341	2.0572

**Case 15:**

**FADD Comparison: 2B/W: 0.1, D/2B: 0.05**

Dia	W	B	2B/W	D/2B	
0.25	50	2.5	0.1	0.05	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (FADD)	K (AFGROW)
0.0475	0.02	1.0038	0.9981	0.7389	0.7420
0.1188	0.05	1.0051	0.9969	0.7913	0.8470
0.2375	0.10	1.0081	0.9944	0.8881	0.9482
0.4750	0.20	1.0177	0.9862	1.0896	1.1204
0.7125	0.30	1.0339	0.9730	1.2551	1.2754
1.1875	0.50	1.0996	0.9250	1.5666	1.5564
1.6625	0.70	1.2704	0.8313	1.9078	1.8541
2.1375	0.90	1.9729	0.6535	2.3960	2.5217
2.2800	0.96	3.0266	0.5666	2.6322	3.4500
2.3275	0.98	4.2396	0.5262	2.8012	4.5293

**Case 16:**

**FADD Comparison: 2B/W: 0.1, D/2B: 0.1**

Dia	W	B	2B/W	D/2B	
0.25	25	1.25	0.1	0.1	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (FADD)	K (AFGROW)
0.0225	0.02	1.0138	0.9935	0.6414	0.6416
0.0563	0.05	1.0161	0.9915	0.7499	0.7664
0.1125	0.10	1.0208	0.9876	0.7916	0.8458
0.2250	0.20	1.0342	0.9766	0.8980	0.9456
0.3375	0.30	1.0546	0.9606	0.9959	1.0338
0.5625	0.50	1.1315	0.9077	1.2045	1.2059
0.7875	0.70	1.3208	0.8121	1.4334	1.4079
1.0125	0.90	2.0740	0.6326	1.8395	1.8878
1.0688	0.95	2.8672	0.5557	2.0860	2.3404
1.1025	0.98	4.4750	0.4851	2.3118	3.2271

**Case 17:**

**FADD Comparison: 2B/W: 0.1, D/2B: 0.5**

Dia	W	B	2B/W	D/2B	
0.25	5	0.25	0.1	0.5	
C	C/(B-D/2)	F <sub>w</sub>	F <sub>offset</sub>	K (FADD)	K (AFGROW)
0.0025	0.02	1.4227	0.8666	0.3194	0.3502
0.0063	0.05	1.4361	0.8608	0.4769	0.5195
0.0125	0.10	1.4604	0.8506	0.6194	0.6699
0.0250	0.20	1.5174	0.8286	0.7672	0.8213
0.0375	0.30	1.5892	0.8040	0.8347	0.9080
0.0625	0.50	1.8054	0.7463	0.9670	1.0413
0.0875	0.70	2.2405	0.6711	1.0408	1.2185
0.1125	0.90	3.7388	0.4999	1.1823	1.5679
0.1188	0.95	5.2414	0.3838	1.2664	1.7004
0.1225	0.98	8.2449	0.2557	1.3325	1.7903